## TIME: 3:00 Hrs

M.M. : 80

General Instructions:
(i) All questions are compulsory.
(ii) The question paper contains two parts A and B .Each part is compulsory .Part A carries 24 marks and Part B carries 56 marks.
(iii) Part A has objective type questions and part B has descriptive type questions
(iv) Part A consists two sections, section I and section II . Section I comprises of 16 very short answer type questions and section II contains 2 case studies. Each case study comprises of 5 case based MCQs. An examinee is to attempt any 4 out of 5 MCQs
(v) Section B comprises of three sections III(2 marks each question),IV (3 marks each question), V (5 marks each question)
(vi) There is no overall choice. However, an internal choice has been provided in questions. You have to attempt only one of the alternatives in all such questions.

## PARTA

|  | Section I |  |
| :---: | :---: | :---: |
| 1. | If A is a skew symmetric matrix of $3 \times 3$ then find the value of $\|A\|$ | 1 |
| 2. | If A is a matrix of order 3 and if $\|A\|=8$ then find the value of $\|3 A\|$ | 1 |
| 3. | If R is a relation and defined by $\mathrm{R}=\left\{\left(\mathrm{a}, \mathrm{a}^{3}\right), \mathrm{a}\right.$ is a prime number less than 5$\}$. Find the element of relation R | 1 |
| 4. | If $\tan ^{-1} x+\tan ^{-1} y=\pi / 4, x y<1$ then find the value of $x+y+x y$ | 1 |
| 5. | If $\mathrm{y}=\log \left(\cos e^{x}\right)$ then find $\frac{d y}{d x}$. | 1 |
| 6. | Evaluate $\int \frac{d x}{\sin ^{2} x \cos ^{2} x}$ | 1 |
| 7. | Find the value of $\int_{-\pi / 2}^{\pi / 2} x^{2} \sin x d x$ | 1 |
| 8. | find the value of $a$ and $b$ if $(i+3 j+9 k) x(3 i-a j+b k)=0$ | 1 |
| 9. | Find order and degree of differential equation $y=x \frac{d y}{d x}-\frac{2}{d y / d x}$ <br> Or <br> Find the sum of order and degree of differential equations $\frac{d^{2} y}{d x^{2}}=\sqrt{1-\left(\frac{d y}{d x}\right)^{2}}$ | 1 |
| 10. | Two independent events $A$ and $b$ are given such that $P(A)=0.3$ and $P(B)=0.6$, find $\mathrm{P}(\mathrm{A}$ and not B$)$ | 1 |


| 11. | Prove that the function $f(x)=3 x^{2}+36 x+5$ is strictly increasing on $R$ OR <br> Prove that the function given by $f(x)=\cos x$ is strictly decreasing in $(0, \pi)$ | 1 |
| :---: | :---: | :---: |
| 12. | If E and F are independent events then prove that E and F ' are also independent. | 1 |
| 13. | If $\mathrm{y}=\mathrm{x}^{\mathrm{x}}$ find dy/dx | 1 |
| 14. | Find the principal value of $\cos ^{-1}(\cos 13 \pi / 6)$ | 1 |
| 15. | Check whether the function $f: R \rightarrow R$ defined as $f(x)=x^{3}$ is one one? | 1 |
| 16. | A relation R in the set of real numbers R defined as $\mathrm{R}=\{(\mathrm{a}, \mathrm{b}) \sqrt{a}=\mathrm{b}\}$ is a function or not? Justify your answer. | 1 |
|  | Section II Both the Case study based questions are compulsory. Attempt any 4 sub parts from each question 17 and 18. |  |
| 17. | An architect designs a building for a multi-national company. The floor consists of a rectangular region with . semicircular ends having a perimeter of 200 m as shown below: |  |
|  | Based on the above information answer the following |  |
| (i) | If $x$ and $y$ represents the length and breadth of the rectangular region, then the relation between the variables is <br> a) $x+\pi y=100$ <br> b) $2 x+\pi y=200$ <br> c) $\pi x+y=50$ <br> d) $x+y=100$ | 1 |
| (ii) | The area of the rectangular region A expressed as a function of x is <br> a) $2 / \pi\left(100 x-x^{2}\right)$ <br> b) $1 / \pi\left(100 x-x^{2}\right)$ <br> c) $x / \pi(100-x)$ <br> d) $\pi y^{2}+2 / \pi\left(100 x-x^{2}\right)$ | 1 |
| (iii) | The maximum value of area A is <br> a) $\pi / 3200 \mathrm{~m}^{2}$ <br> b) $3200 / \pi m^{2}$ <br> c) $5000 / \pi m^{2}$ <br> d) $1000 / \pi \mathrm{m}^{2}$ | 1 |


| (iv) | The CEO of the multi-national company is interested in maximizing the area of the whole floor including the semi-circular ends. For this to happen the value of $x$ should be <br> a) 0 m <br> b) 30 m <br> c) 50 m <br> d) 80 m | 1 |
| :---: | :---: | :---: |
| (v) | The CEO of the multi-national company is interested in maximizing the area of the whole floor including the semi-circular ends. For this to happen the value of x should be <br> a) 0 m <br> b) 30 m <br> c) 50 m <br> d) 80 m | 1 |
| 18. | In an office three employees Vinay, Sonia and Iqbal process incoming copies of a certain form. Vinay process $50 \%$ of the forms. Sonia processes $20 \%$ and Iqbal the remaining $30 \%$ of the forms. Vinay has an error rate of 0.06 , Sonia has an error rate of 0.04 and Iqbal has an error rate of 0.03 Based on the above information answer the following | 1 |
| (i) | The conditional probability that an error is committed in processing given that Sonia processed the form is : <br> a) 0.0210 <br> b) 0.04 <br> c) 0.47 <br> d) 0.06 | 1 |
| (ii) | The probability that Sonia processed the form and committed an error is:a) 0.005 b) 0.006 c) 0.008 d) 0.68 | 1 |
| (iii) | The total probability of committing an error in processing the form is <br> a) 0 <br> b) 0.047 <br> c) 0.234 <br> d) 1 | 1 |
| (iv) | The manager of the company wants to do a quality check. During inspection he selects a form at random from the days output of processed forms. If the form selected at random has an error, the probability that the form is NOT processed by Vinay is : <br> a) 1 <br> b) $30 / 47$ <br> c) $20 / 47$ <br> d) $17 / 47$ | 1 |
| (v) | (v)Let A be the event of committing an error in processing the form and let E1, E2 and E3 be the events that Vinay, Sonia and Iqbal processed the form. The value of $\sum_{i=1}^{3} P\left(E_{i} / \mathrm{A}\right)$ is a) 0 <br> b) 0.03 <br> c) 0.06 <br> d) 1 | 1 |
|  | $\begin{gathered} \text { Part B } \\ \text { Section III } \\ \hline \end{gathered}$ |  |
| 19. | Prove that $\cos ^{-1} \frac{12}{13}+\sin ^{-1} \frac{3}{5}=\sin ^{-1} \frac{56}{65}$ | 2 |
| 20. | If $\mathrm{x} \cos (\mathrm{a}+\mathrm{y})=\cos y$ with $\mathrm{a} \neq \pm 1$ then prove that $\frac{d y}{d x}=\frac{\cos ^{2}(a+y)}{\sin a}$ Or if $x=a(1-\cos t), y=(t+\sin t)$ find $d y / d x$ | 2 |
| 21. | If $f: R \rightarrow R$ is the function defined by $f(x)=4 x^{3}+7$ then show that $f$ is bijective | 2 |
| 22. | If $\vec{a}+\vec{b}+\vec{c}=\mathbf{0}$ and $\|\vec{a}\|=5,\|\vec{b}\|=6$ and $\|\vec{c}\| 9$, then find angle between $\vec{a}$ and $\vec{b}$. | 2 |
| 23. | Find the vector equation of the line passing through the point $\mathrm{A}(1,2,-1)$ and parallel to the line $5 x-25=14-7 y=35 z$. | 2 |


| 24. | Prove that $f(x)=2 x-\|x\|$ is not differentiable at $\mathrm{x}=0$ | 2 |
| :---: | :---: | :---: |
| 25. | Solve the differential equation $\frac{d y}{d x}=x^{3} \operatorname{cosec} y$ given that $y(0)=0$ | 2 |
| 26. | Find the area of the parallelogram ABCD whose side and diagonal AC are given by the vectors $3 \mathrm{i}-\mathrm{j}+4 \mathrm{k}$ and $4 \mathrm{i}+5 \mathrm{k}$ respectively. | 2 |
| 27. | If $\mathrm{A}=\left[\begin{array}{cc}3 & 1 \\ -1 & 2\end{array}\right]$ Show that $\mathrm{A}^{2}-5 \mathrm{~A}+7 \mathrm{I}=0$ Hence find $\mathrm{A}^{-1}$ | 2 |
| 28. | Evaluate $\int_{0}^{1} x(1-x)^{n} d x$ | 2 |
|  | Section IV |  |
| 29. | Evaluate $\int \frac{\sec ^{2} x}{3+\tan ^{2} x} d x \quad$ or $\int \frac{1}{x^{2}+4 x+8} d x$ | 3 |
| 30. | Solve the differential equation $\mathrm{xdy}-\mathrm{ydx}=\sqrt{x^{2}+y^{2}} \mathrm{dx}$ | 3 |
| 31. | If $\vec{\alpha}=3 \hat{\imath}-\hat{\jmath}$ and $\vec{\beta}=2 \hat{\imath}+\hat{\jmath}-3 \hat{k}$, then express $\vec{\beta}=\overrightarrow{\beta_{1}}+\overrightarrow{\beta_{2}}$, where $\overrightarrow{\beta_{1}}$ is parallel to $\vec{\alpha}$ and $\overrightarrow{\beta_{2}}$ is perpendicular to $\vec{\alpha}$ | 3 |
| 32. | If the function f is defined as $f(x)=\left\{\begin{array}{cl}a \sin \frac{\pi}{2}(x+1) & , x \leq 0 \\ \frac{\tan x-\sin x}{x^{3}}, & x>0\end{array}\right.$ is continuous at $\mathrm{x}=0$. Prove that $\mathrm{a}=\frac{1}{2}$ | 3 |
| 33 | Check whether the relation $R$ in the set $Z$ of integers defined as $R=\{(a, b): a+b$ is divisible by $2\}$ is reflexive, symmetric and transitive? | 3 |
| 34 | If $0<x<\pi / 2$ and $y=\cot ^{-1}\left(\frac{\sqrt{1+\sin \mathrm{x}}+\sqrt{1-\sin \mathrm{x}}}{\sqrt{1+\sin \mathrm{x}}-\sqrt{1-\sin \mathrm{x}}}\right)$, find $\frac{\mathrm{dy}}{\mathrm{dx}}$ | 3 |
| 35. | Evaluate $\int_{0}^{\frac{\pi}{2}} \log \sin x d x$ | 3 |
|  | Section V |  |
| 36. | If the lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$ intersect then find the value of k and hence find the equation of the plane containing these lines <br> OR <br> Find the foot of the perpendicular drawn from the point $(-1,3,-6)$ to the plane $2 x+y-2 z+5=0$ .Also find the equation and length of the perpendicular | 5 |
| 37. | A wire of length 34 m is to be cut into two pieces. One of the piece is to be made into a square and the other into a rectangle whose length is twice its breadth. W hat should be the lengths of the two pieces, so that the combined area of the square and the rectangle is minimum. | 5 |


| 38. | A dealer deals in two items A and B.He has Rs 15000 invest and a space to store atmost 80 pieces .Item A cost him Rs 300 and item B cost him Rs 150 .He can sell items A and B at profit of Rs 40 and Rs 25 respectively .assuming that he can sell all that he buys formulate as a linear programming problem for maximum profit band solve graphically. <br> Or <br> Solve the following linear programming problem graphically <br> Maximise $Z=3 x+9 y$,subject to the constraints: $\begin{gathered} x+3 y \leq 60 \\ x+y \geq 10 \\ x \leq y \\ x \geq 0, y \geq \end{gathered}$ | 5 |
| :---: | :---: | :---: |

